MEASUREMENT OF THE ACOUSTIC ADMITTANCE OF THE BURNING SURFACE OF GUNPOWDER

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Two methods of measuring the acoustic admittance of the burning surface of gunpowder, namely, the method of critical conditions and that of the varying surface, are developed.

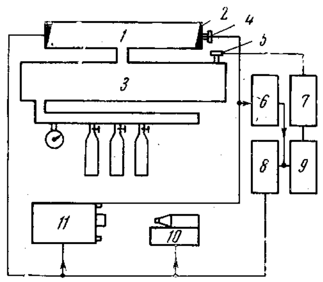
The method of critical conditions is based on the measurement of the limit of self-excitation of unstable combustion in resonators of simple form whose acoustic losses can be accurately determined.

The method of the varying surface consists in measuring the rate of increase or decrease of the amplitude of oscillations in a T-chamber during the combustion of a gunpowder sample whose burning surface varies with time.

The dependence of acoustic admittance on oscillation frequency and pressure are investigated. Both methods are applicable in a wide range of frequencies.

The acoustic admittance of a burning surface is a fundamental physical property which defines the tendency of gunpowder to acoustic instability of combustion. Under conditions of acoustic instability of combustion of condensed systems, pressure waves are usually intensified at the burning surface of gunpowder in the narrow zone of intense chemical reactions. The characteristic dimensions of this zone are small in comparison with those of the resonance volume and of the wavelength. Variation of the acoustic wave parameters produced by the interaction with the combustion process is characterized by the coefficient of wave refraction from the burning surface of gunpowder. The acoustic admittance of a burning surface is defined as the ratio of variation of the rate of combustion products outflow $\delta\vartheta$ from the gunpowder surface to that of pressure δp in the sound wave at the burning surface

$$\zeta = \delta\vartheta / \delta p$$



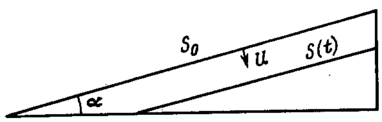


Fig. 1. Block diagram of [experimental] setup: 1) T-chamber; 2) gunpowder samples; 3) receiver; 4) pressure oscillation sensor; 5) mean pressure sensor; 6) magnetic recorder; 7) amplifier; 8) automatic control unit; 9) loop oscillograph; 10) tape winder; 11) electron oscillograph.

Fig. 2. Diagram of the gunpowder sample.

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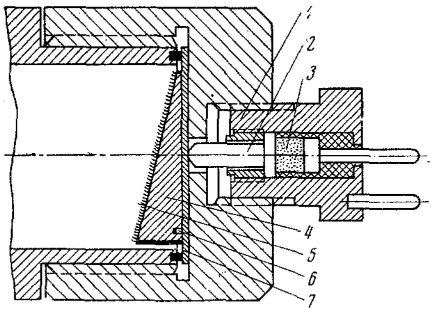


Fig. 3. Diagram of the pressure fluctuation sensor and of the [method of] mounting of the gunpowder sample: 1) sensor casing; 2) wave guide; 3) piezoceramic element; 4) gunpowder sample; 5) igniting compound; 6) lead azide crystal; and 7) membrane.

The real Re ζ and the imaginary Im ζ parts of acoustic admittance define, respectively, the absolute value of the reflection coefficient and the phase shift owing to reflection.

This paper deals with experimental determination of the acoustic admittance of the burning surface of gunpowder. The methods of critical conditions and of varying surface are developed and applied to the measurement of acoustic admittance of a burning surface.

Self-excitation of an auto-oscillating system occurs when the input of energy exceeds its losses. Taking into consideration the acoustic energy balance of a resonator of the simplest form permitting a reliable calculation of acoustic losses, and with the knowledge of the limit of stable combustion, it is possible to determine the acoustic admittance of a burning surface. Two variants of the method of critical conditions are given below.

Investigations of the instability of combustion of nitroglycerin powder A (caloric value Q = 1100 kcal/kg) samples with a bore had shown that the excitation of longitudinal oscillations of combustion products in the channel is considerably affected by the ratio of the length of the sample to its bore diameter L/d. At high L/d longitudinal oscillations of combustion products are always present, while for $L/d \leq 14$ such oscillations were never excited.

If in the study of the amplitude variation of a plane wave propagating in a channel with burning walls, having its dimensionless acoustic admittance $\sigma = \zeta\rho_1 c_1$, where ρ_1 is the density of products of combustion and c_1 the speed of sound in the latter, we assume that sound is radiating only from the open side of the channel, the conditions for intensification of longitudinal oscillations can, to within magnitudes of the first order of smallness, be written as

$$z - L\varkappa/d > 0 \quad \varkappa = \mathrm{Re}\,\sigma \ll 1 \,. \tag{1}$$

Here z is the dimensionless acoustic impedance of the channel open end.

In the case of an open end with a wide flange, using the known equations of acoustics [1], we obtain for

$$z = \frac{\pi^2 n^2 d^2}{8L^2}, \tag{2}$$

where n is the order of the longitudinal oscillation harmonic. Thus the condition of intensification of natural longitudinal oscillations in a channel with burning walls is

$$\frac{\pi^2 n^2 d^2}{8L^2} + \frac{\varkappa L}{d} < 0, \tag{3}$$

if sound attenuation in the volume of combustion products is neglected. It will be seen from (3) that intensification of oscillations is possible when $\varkappa < 0$, which can occur at the burning surface.

For $\varkappa < 0$ and $(L/d)^2 > \frac{1}{8}\pi^2 n^2/|\varkappa|$ increase of the oscillation amplitude, owing to intensification at the burning surface, exceeds the attenuation produced by radiation from the open ends and, consequently, the oscillations may become self-excited.

Taking L/d = 14 as the critical value and assuming n = 2 (experimental conditions), from equation (3) we find that for gunpowder A at pressures of 50-100 kgf/cm^2 and a frequency of 10 kHz (experimental conditions) $|\varkappa| \approx 10^{-3}$. The accuracy of these experiments was such that only the order of magnitude of $\varkappa$ was determined. The obtained $|\varkappa| \approx 10^{-3}$ corresponds to a coefficient $R \approx 1.002$ of reflection from the burning surface of gunpowder and, as regards its order of magnitude, coincides with the theoretical estimate.

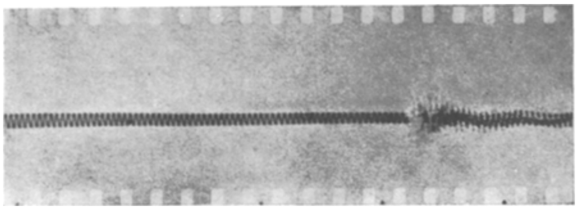


Fig. 4. A section of the oscillogram used in calculations.

This is the result of the first experimental determination of the acoustic admittance of the burning surface of gunpowder. (The method of critical conditions was proposed in 1960. Measurements were carried out in 1961. The results presented here have been taken from the 1960 and 1961 Reports of the Institute of Chemical Physics of the Academy of Sciences of the USSR.)

Another application of the method of critical conditions consists in the experimental determination of $\varkappa$ by using two plane-parallel plates of gunpowder samples. During the combustion of such samples in the cavity between the two plane-parallel plates of gunpowder and rigid lateral walls, oscillations of the products of combustion normal to the faces of gunpowder plates were induced at a frequency of about 50 kHz (the fundamental mode). The critical condition for self-excited oscillations was found to be the achievement of a certain thickness of the gunpowder plate[s]. The simple geometry of the used sample makes possible a sufficiently accurate determination of sound energy losses at the instant of self-excitation of oscillations, and to determine the unknown $\varkappa$ from the critical conditions for the occurrence of such oscillations. The fundamental acoustic losses are in this case due to the reflection of sound waves from the gunpowder plates. Sound absorption by the gaseous phase, in the absence of solid particles in it, may be neglected, owing to its smallness [2]. The coefficient of radiation of transverse oscillations of gas in the cavity, emanating from its rectangular open ends, is close to zero [1]. The metal walls are considered to be rigid, and, consequently, only the sound energy losses due to the reflection of oscillations from the gunpowder plates, and the energy input generated by wave intensification at the burning surface of gunpowder are taken into account in calculations.

The condition for self-excitation of oscillations is written as $|R| \geq 1$, where R is the complex coefficient of sound wave reflection from the burning surface of gunpowder. Expressing R in terms of σ and of the acoustic admittance of the non-burning gunpowder plate, taking into consideration the absorption of sound by the gunpowder, and using equations of acoustics [3], we obtain

$$\varkappa = \frac{z_1}{z_2}\left\{\frac{z_1 z_2\,[(A^2+B^2+C^2)(1+\varepsilon)-2\varepsilon^2(B^3+C)]}{2z_1z_2(\varepsilon AC+AB)-z_1^2(1+\varepsilon^2)A^2+z_2^2(B^2+C)} + \frac{z_1^2(1+\varepsilon^2)(\varepsilon AC-AB)-z_2^2(\varepsilon AC+AB)}{2z_1z_2(\varepsilon AC+AB)-z_1^2(1+\varepsilon^2)A^2+z_2^2(B^2+C^2)}\right\};$$

$$A = \exp(-4\delta_2 b) + 2\exp(-2\delta_2 b)\cos k_2 b + 1,$$
$$B = \exp(-4\delta_2 b) - 1, \qquad C = 2\exp(-2\delta_2 b)\sin 2k_2 b,$$
$$z_1 = \rho_1 c_1, \quad z_2 = \rho_2 c_2, \quad k_2 = 2\pi f / c_2, \quad \varepsilon = \delta_2 / k_2. \qquad (4)$$

Here ρ_1 and ρ_2 are, respectively, the densities of the products of combustion and of the condensed phase of gunpowder; c_1 and c_2 are the speed of sound in the products of combustion and in the material of gunpowder, respectively; δ_2 is the coefficient of sound absorption in the gunpowder; f is the frequency of oscillations at the instant of excitation; and b is the thickness of the gunpowder plate at the instant of excitation. Substituting into equation (4) $z_1 = 4.75 \cdot 10^2$ g/cm^2 and $\delta_2 = 0.4$ cm^{-1}, and the calculated from experimental data values of $z_2 = 2.55 \cdot 10^5$ g/cm^2 sec, $k_2 = 1.97$ cm^{-1}, and b = 0.8 cm, for gunpowder A we obtain $\varkappa = 0.6 \cdot 10^{-3}$ at a frequency of 50 kHz and pressure of 40 kgf/cm^2.

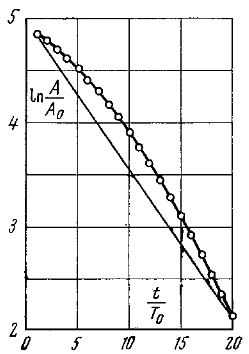


Fig. 5. Dependence of the logarithm of the relative amplitude of oscillations on time (T_0 is the period of oscillations).

Since sound attenuation in the bulk of combustion products containing solid particles cannot be neglected, the described method of determination of $\varkappa$ was used in investigations of gunpowders whose products of combustion were free of condensed particles. This method permits the investigation of the acoustic admittance of a burning surface at extremely high frequencies (50-100 kHz), unobtainable by other methods.

Transient characteristics of the oscillation process generated in the gunpowder combustion chamber may be used for the determination of σ, as was done by Horton [4-6]. These characteristics depend on σ and on the cumulative attenuation of sound in the experimental combustion chamber. Horton's method and its variants consist in burning gunpowder samples with a constant time-independent area of the burning surface in a T-shaped chamber, and σ is calculated from two different measurements of the transient characteristics of the oscillation process corresponding to different conditions of combustion. Measurements are made right at the beginning and/or at the end of combustion of the sample. Results of such measurements are inevitably affected by the ignition and burn-out phases of gunpowder combustion.

A distinctive feature of the method of variable surface described below is the use of gunpowder samples whose burning surface area varies with time. If the area of the burning surface of the sample varies to a known law S(t), the unknown σ of the gunpowder can be determined, as shown below, by a single measurement of the transient characteristics of products of combustion induced in a T-chamber or any other experimental equipment. The ignition and burn-out processes of the sample do not in this case affect the results of experiments, since, generally speaking, measurements are made in the middle of the gunpowder combustion process.

Two experimental setups for use with the method of varying [combustion] surface were produced and tested. The first had its combustion chamber in the shape of a resonance space consisting of two Helmholtz resonators interconnected by throats and containing samples of gunpowder whose burning surface area increased linearly with time. This setup is suitable for determining σ at low frequencies (below 1000 Hz).

Fundamental results were obtained on the setup shown diagrammatically in Fig. 1. A T-shaped chamber containing samples of gunpowder serves as the resonance space; it is connected to a large-volume chamber, which before an experiment is filled with nitrogen at the required pressure. The gunpowder sample is in the shape of a wedge with the acute angle equal 13°, a base of 38 mm, width a = 28 mm, and a height of 10 mm (Fig. 2). The samples are glued to brass membranes, and then rigidly attached to the chamber end-walls. The sides of samples were armor-plated and their front faces covered with an explosive compound. With the progress of combustion the area of the burning surface of the sample decreased according to equation

$$S(t) = S_0 - 2a\, Ut \operatorname{ctg} 2\alpha \tag{5}$$

where S_0 is the area of the burning surface and U the rate of combustion.

Pressure fluctuations induced in the resonator by the combustion of gunpowder are picked up by the piezoelectric sensor whose signals are recorded on an electron oscillograph with a high-speed mechanical unrolling device. A diagram of the sensor and of the [method of] sample mounting is shown in Fig. 3. Pressure fluctuations in the T-chamber were, also, recorded on a magnetic tape whose signals were at the same time passed to a loop oscillograph. The mean pressure in the chamber was also recorded. Ballistic gunpowders of mark H (Q ≈ 870 kcal/kg) and of mark A (Q ≈ 1100 kcal/kg) were used in these experiments.

The effective specific acoustic admittance of the T-chamber end-face (per unit of its area) is proportional to the area of that part of the face which is taken up by the burning gunpowder

$$Y_{eff} = \frac{S(t)}{S_a}\zeta = \frac{\langle \delta v \rangle}{\delta p} . \tag{6}$$

Here σp is the pressure in the acoustic wave at the boundary, $\langle \delta \vartheta \rangle$ is the acoustic speed averaged over the whole of the end-face, and S_a is the area of the end-face.

Walls of the T-chamber, not taken up by the gunpowder, are considered as being rigid. The expression (6) is used as the boundary condition for solving the wave equation defining the oscillations of combustion products, with the attenuation taken into account by

$$\partial^2\psi / \partial t^2 + \beta\partial\psi / \partial t = c_1^2\partial^2\psi / \partial x^2, \quad (7)$$

where β is a function dependent on frequency and characterizing the absorption of energy, and ψ is the gas particle displacement whose expression will be sought in the form

$$\psi = \sum_{n=1}^{\infty} A_n\varphi(x)\exp(\alpha_n + j\omega_n)t,$$

Here A_n is the amplitude, α_n a complex constant, and ω_n is the angular frequency in the absence of attenuation or intensification of sound.

Equation (7) is solved on the usual assumptions of the T-chamber theory [4, 6], i.e.:

1. Only longitudinal oscillations are taken into consideration.

2. Variations of the chamber length is disregarded, since the wave length of the first harmonic of longitudinal oscillations generated in the T-chamber is considerably greater than the sample thickness.

3. The effect of the mean flow is disregarded, because in this case (in which the acoustic boundary layer is thinner than the hydrodynamic) the coefficient of sound attenuation in a cavity with a stream of gas is virtually independent of the stream velocity, as shown theoretically and by experiment [7, 8].

4. Acoustic losses are assumed constant throughout the time of measurement (which is small in comparison with the total time of combustion).

5. Gaseous products [of combustion] are considered to be homogeneous with an average density ρ_1 and a speed of sound c_1. The process is, furthermore, considered to be quasi-stationary, since the intrinsic time of variation of the burning surface of gunpowder considerably exceeds the oscillation period.

The boundary conditions are

$$\frac{S}{S_a}\zeta = \frac{\langle\delta v\rangle}{\delta p} = \frac{1}{\rho_1 c_1^2}\frac{\partial\psi/\partial t}{\partial\psi/\partial x} \text{ for } x=0,\ x=l\ . \quad (8)$$

Here l is the length of the chamber.

Using the method of consecutive approximations and taking into account the quasi-stationarity of the process, we find the solution of Eq. (7) for the first harmonic

$$\psi = A_0\exp\left\{\frac{4c_1}{l}\frac{aU\,\mathrm{Re}\,(\rho_1c_1\zeta)}{S_a\,\mathrm{tg}\,2\alpha}t^2 - \left[\frac{\beta}{2} + 2\frac{c_1}{l}\mathrm{Re}\,(\rho_1c_1\zeta)\frac{S_0}{S_a}\right]t + j\frac{c_1}{l}\left[\pi - 2\mathrm{Im}\,(\rho_1c_1\zeta)\left(\frac{S_0}{S_a} - \frac{2aUt}{S_a\,\mathrm{tg}\,2\alpha}\right)\right]t\right\}, \quad (9)$$

The real part of this expression defines the variation of the oscillation amplitude with respect to time, and can be written as

$$A_t(t) = A_0\exp(Dt^2 - Gt), \quad D = 4\frac{c_1}{l}\frac{aU\varkappa}{S_a\,\mathrm{tg}\,2\alpha}, \quad (10)$$
$$G = \frac{\beta}{2} + 2\frac{c_1}{l}\varkappa\frac{S_0}{S_a}, \quad \varkappa = \mathrm{Re}\,(\rho_1c_1\zeta) = \mathrm{Re}\,\sigma\ .$$

The coefficient D at t^2 depends only on the unknown $\varkappa$ and on the a priori known or experimentally determined parameters of the process and equipment. Thus the problem of the determination of $\varkappa'$ reduces to finding the coefficients of the parabola defining the dependence of $\ln A_t$ on time, which is determined experimentally. Then

$$\varkappa = \frac{DS_a \operatorname{tg} 2\alpha}{4(c_1/l)\,Ua}. \qquad (11)$$

The [functional] dependence $\ln A_t(t)$ was determined from the high-speed recording of oscillations. A section of the oscillogram used in calculations is shown in Fig. 4.

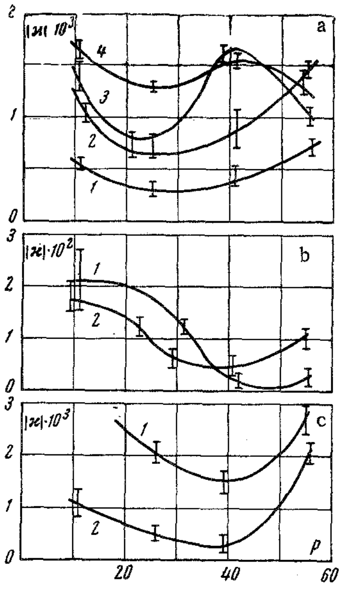
The burst on the oscillogram relates to the instant of explosion of the lead azide crystal mounted in the base of the sample (Fig. 3) for the purpose of separating in the oscillogram the section of gunpowder combustion from that corresponding to oscillation attenuation on completion of combustion. The dependence $\ln A_t = \varphi(t)$ determined in one of the experiments is shown in Fig. 5.

The coefficient D at t^2 in the experimentally determined dependence $\ln A_t = \varphi(t)$ was calculated by the method of least squares. At least 20 experimental readings were used in the calculation. Maintenance of the law of linear decrease of the burning surface area was checked throughout the time of combustion by extinguishing the burning of gunpowder samples at various stages of combustion.

Measurements of the acoustic admittance of the burning surface of investigated gunpowders were made at pressures of 10, 25, 40, and 55 kgf/cm^2 in the range of frequencies from 800 to 5000 Hz. Experimental results are shown in Figs. 6 and 7, where the thin vertical lines indicate the maximum scatter of experimental readings. Fig. 6 a shows the dependence of the real part of the dimensionless acoustic admmittance ($|\varkappa| = |\mathrm{Re}\sigma|$) on pressure at various frequencies for mark H gunpowder.

Dependence of the absolute value of $\varkappa$ on pressure for the same frequencies for the high-calorific-value gunpowder A is given in Fig. 6 b and c.

The experimentally obtained values of the real part of σ are negative, which indicates an intensification of acoustic waves. It is clear that in the investigated range of frequencies and pressures the absolute value of $\varkappa$ is around 10^{-3}-10^{-2}, which is in good agreement with the theoretical and experimental estimates obtained earlier. These results also show that $\varkappa$ is greater for the high-calorific-value fast-burning mark A powder than for the comparatively slower burning mark H. Theoretical investigations [9, 10] show



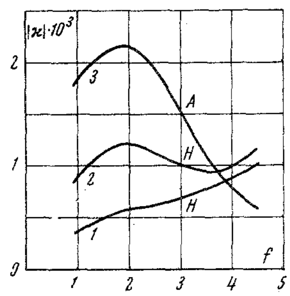


Fig. 6. Dependence of $|\varkappa'|$ on pressure p in kgf/cm^2. a) Curves 1, 2, 3, and 4 are for gunpowder H and relate to frequencies f = 0.8, 1.8, 2.9, and 4.9 kHz; b) curves 1 and 2 are for gunpowder A and relate to f = 0.8 and 1.8 kHz, respectively; c) curves 1 and 2 are for gunpowder A and relate to f = 2.9 and 4.9 kHz, respectively.

Fig. 7. Dependence of $|\varkappa|$ on frequency f in kHz for the A and H gunpowders. Curves 1, 2, and 3 relate to [pressures] p = 25, 55, and 40 kgf/cm^2, respectively.

that the calorific value and the rate of combustion of gunpowder substantially affect the acoustic admittance of its burning surface.

The minima of curves illustrating the experimentally obtained dependence of $|\varkappa|$ on p at various frequencies, shown in Fig. 6, should be noted. These minima for gunpowder H occur in the pressure range of 20-30 kgf/cm^2, while for that of mark A they appear in the neighborhood of 40 kgf/cm^2.

Figure 1 shows $|\varkappa|$ as a function of frequency at constant pressure.

The most interesting property of these curves is the presence of smooth maxima of $|\varkappa|$ at frequencies of 1.5-2 kHz for gunpowders A and H at pressures of 40 and 55 kgf/cm^2, respectively. This "preferred" frequency is probably determined by the relation between the oscillation frequency and the characteristic combustion time of a gunpowder. Comparison of experimental and theoretical data is difficult owing to the highly idealized assumptions made in all theories of the acoustic instability of combustion. Thus, for example, the experimentally observed slow rate of increase of $|\varkappa|$ with increasing frequency is in the case of gunpowder H in good correlation with the theoretical predictions in [9], which cannot be said of the results obtained with gunpowder A.

A fuller and better substantiated comparison of experimental and theoretical data would necessitate measurements in a wider range of frequencies, as well as the theoretical investigation of various models of combustion. The described method can be actually extended to cover the frequency range of from 300-10,000 Hz. A detailed study of the dependence of σ on frequency would require individual measurements to be taken in considerably reduced frequency intervals.

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